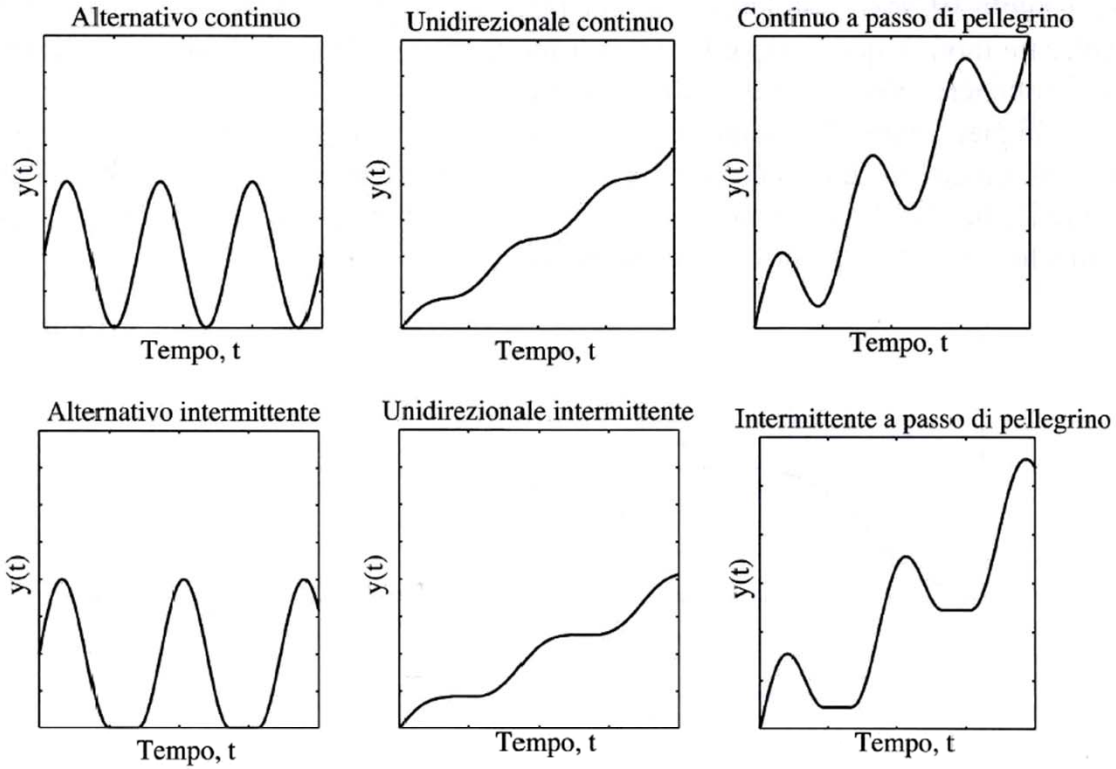
MECCANICA DEGLI AZIONAMENTI

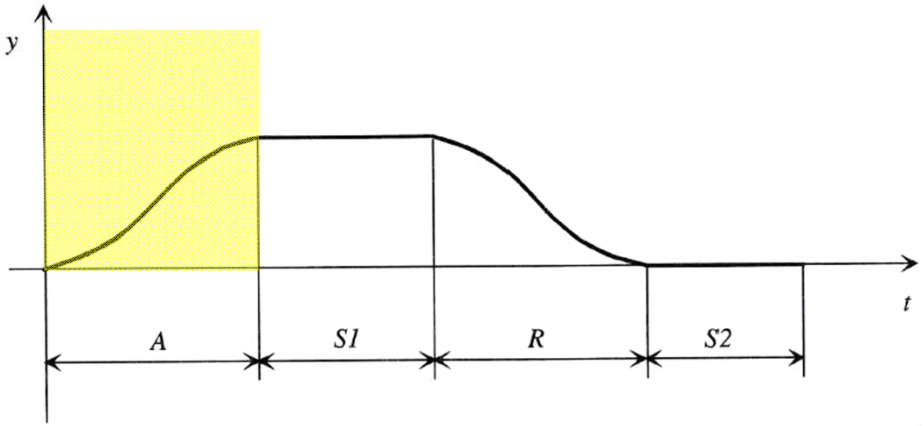
Presentazione07: Leggi di Moto



$$y = y(t) = y(\theta(t)) \quad \rightarrow \quad y(\theta)$$

$$\dot{y}(t) = \frac{dy}{dt} = \frac{d\theta}{dt}\frac{dy}{d\theta} = \omega\, y'$$

$$\ddot{y}(t) = \frac{d^2y}{dt^2} = \omega^2 y'' + \dot{\omega}\, y'$$



IL PRESENTE MATERIALE È RISERVATO AL PERSONALE DELL'UNIVERSITÀ DI BOLOGNA E NON PUÒ ESSERE UTILIZZATO AI TERMINI DI LEGGE DA ALTRE PERSONE O PER FINI NON ISTITUZIONALI

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Leggi di Moto

Funzioni di forma

$$y = H\, f(\xi), \quad \xi = \frac{\theta}{\theta_a} = \frac{t}{t_a}$$

$$y' = \frac{H}{\theta_a}\frac{df}{d\xi} = \frac{H}{\theta_a} g(\xi), \quad \dot{y} = \frac{H}{t_a}\frac{df}{d\xi} = \frac{H}{t_a} g(\xi)$$

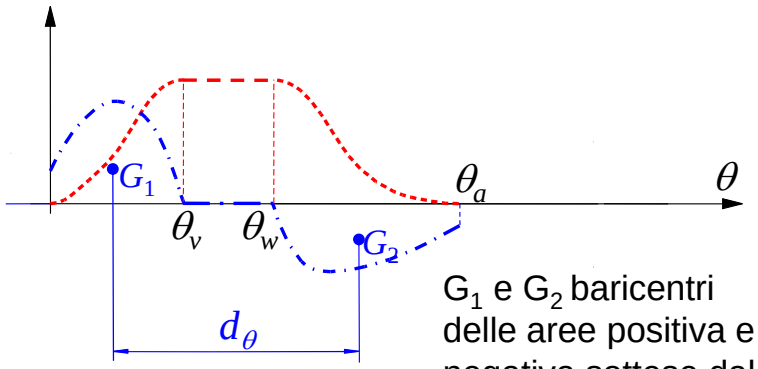
$$y'' = \frac{H}{\theta_a^2}\frac{d^2 f}{d\xi^2} = \frac{H}{\theta_a^2} h(\xi), \quad \ddot{y} = \frac{H}{t_a^2}\frac{d^2 f}{d\xi^2} = \frac{H}{t_a^2} h(\xi)$$

$$y'y'' = \frac{H^2}{\theta_a^3} g(\xi) h(\xi), \quad \dot{y}\ddot{y} = \frac{H^2}{t_a^3} g(\xi) h(\xi)$$

Coefficiente di velocità

$$c_v = g_{\max} = \frac{y'_{\max}}{H/\theta_a} = \frac{\dot{y}_{\max}}{H/t_a} \quad (c_v \geq 1)$$

$$c_v = \frac{\theta_a}{d_\theta}$$



G_1 e G_2 baricentri delle aree positiva e negativa sottese dal diagramma dell'accelerazione

Coefficienti di accelerazione

$$c_{a+} = h_{\max} = \frac{y''_{\max}}{H/\theta_a^2} = \frac{\ddot{y}_{\max}}{H/t_a^2}, \quad c_{a-} = |h_{\min}| = \frac{|y''_{\min}|}{H/\theta_a^2} = \frac{|\ddot{y}_{\min}|}{H/t_a^2}$$

$$c_a = \max\{c_{a+}\,,\, c_{a-}\} = \frac{|y''|_{\max}}{H/\theta_a^2} = \frac{|\ddot{y}|_{\max}}{H/t_a^2} \quad (c_a \geq 4)$$

$$\int_0^{\theta_a} y'' d\theta = 0$$

$$\int_0^{\theta_a} y' d\theta = H$$

$$\int_0^{\theta_a} y'' \theta\, d\theta = -H$$

Leggi di Moto

Forza e potenza delle azioni inerziali sul cedente

$$F_i = -m\ddot{y} = -my''\omega^2, \quad W_i = -m\ddot{y}\dot{y} = -my''y'\omega^3$$

$$\eta = 1 \quad \Rightarrow \quad W_{mov,i} = C_{mov,i}\omega = -W_i = my''y'\omega^3 = m\omega^3 \frac{H^2}{\theta_a^3} g(\xi) h(\xi)$$

$$C_{mov,i} = \frac{W_{mov,i}}{\omega} = m\omega^2 \frac{H^2}{\theta_a^3} g(\xi) h(\xi)$$

Coefficienti di coppia

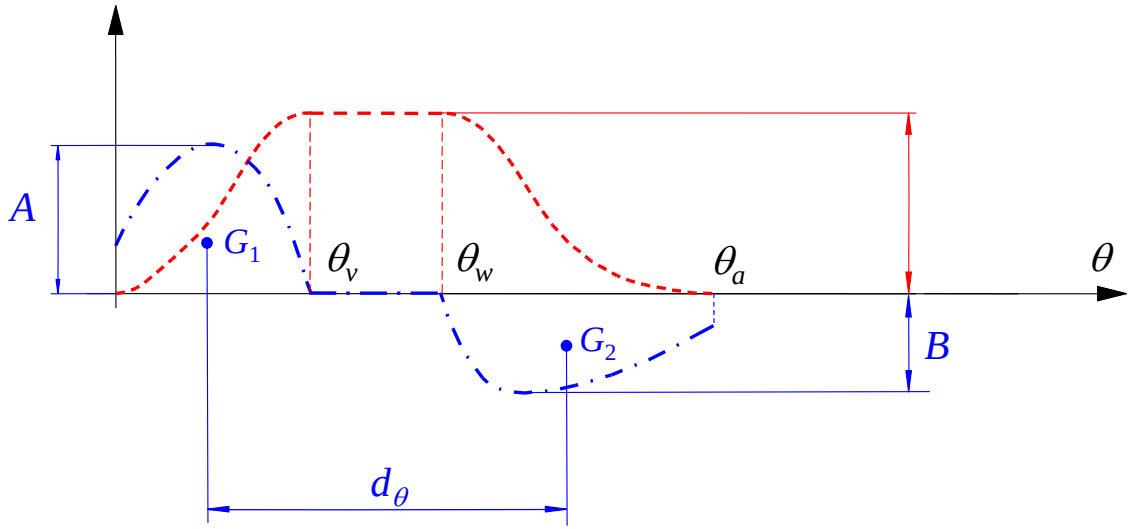
$$c_{k+} = (gh)_{\max} = \frac{(y'y'')_{\max}}{H^2/\theta_a^3} = \frac{(\dot{y}\ddot{y})_{\max}}{H^2/t_a^3}, \quad c_{k-} = \left|(gh)_{\min}\right| = \frac{\left|(y'y'')_{\min}\right|}{H^2/\theta_a^3} = \frac{\left|(\dot{y}\ddot{y})_{\min}\right|}{H^2/t_a^3};$$

$$c_k = \max\{c_{k+}, c_{k-}\} = \frac{|y'y''|_{\max}}{H^2/\theta_a^3} = \frac{|\dot{y}\ddot{y}|_{\max}}{H^2/t_a^3}.$$

$$\left|C_{mov,i}\right|_{\max} = m\omega^2 \frac{H^2}{\theta_a^3} c_k \qquad c_v^2 \le c_k \le c_v c_a$$

Leggi di Moto

G_1 e G_2 baricentri delle aree positiva e negativa sottese dal diagramma dell'accelerazione



$$c_v = \frac{\theta_a}{d_\theta} \quad \Rightarrow \quad y'_{\max} = c_v \frac{H}{\theta_a} = \frac{\cancel{\theta_a}}{d_\theta}\frac{H}{\cancel{\theta_a}} = \frac{H}{d_\theta}$$

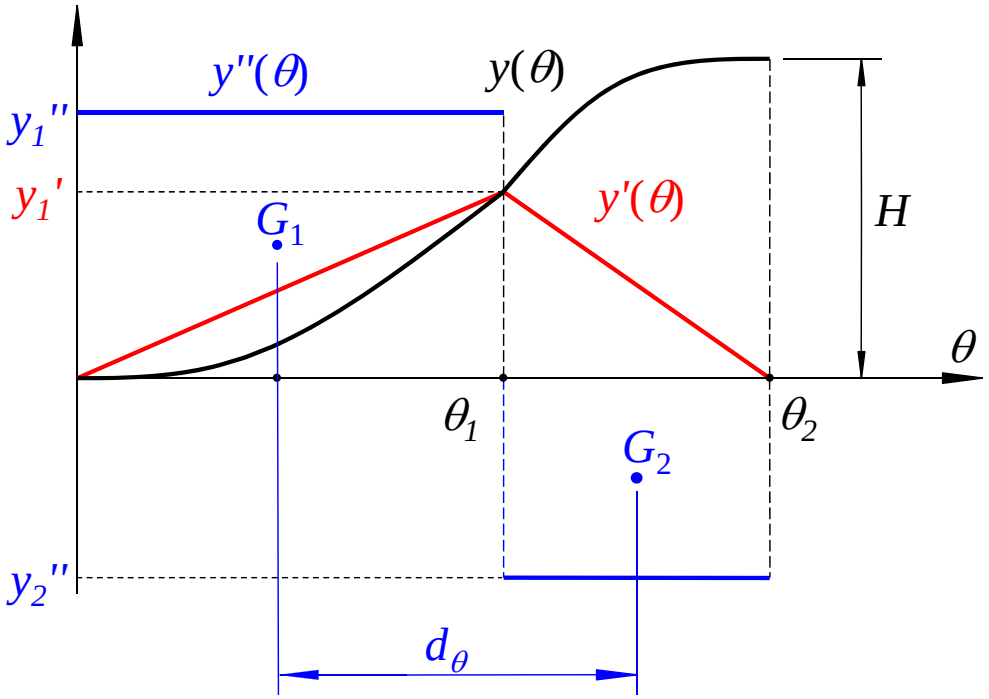
$$S_1 = \int_0^{\theta_v} y'' d\theta = S_1\left(A, \theta_v\right) = y'_{\max} \qquad A \quad \Rightarrow \quad c_{a+} = \frac{A}{H/\theta_a^2}$$

Se y' e $|y''|$ sono massimi 'contemporaneamente' $\quad c_{k+} = c_v c_{a+}, \quad c_{k-} = c_v c_{a-}$

altrimenti $\quad \dfrac{d\left(y'y''\right)}{d\theta} = 0 \quad \Rightarrow \quad \theta \in [0, \theta_v]: \ \left(y'y''\right)_{\max} \quad \Rightarrow \quad c_{k+} = \dfrac{\left(y'y''\right)_{\max}}{H^2/\theta_a^3}$

Leggi di Moto

Legge «*parabolica*»



$$\dot{y} = \Omega\, y'; \quad \ddot{y} = \Omega^2\, y'' \quad (\Omega = \text{cost.})$$

$$d\vartheta = \Omega\, dt$$

$$\dot{y}_2 = \int_0^{t_2} \ddot{y}\, dt = \Omega \int_0^{\vartheta_2} y''\, d\vartheta =$$

$$= \Omega[y_1''\, \vartheta_1 + y_2''\, (\vartheta_2 - \vartheta_1)] = 0$$

$$y_1''\, \vartheta_1 = -y_2''\, (\vartheta_2 - \vartheta_1)$$

$$\dot{y}_1 = \int_0^{t_1} \ddot{y}\, dt = \Omega \int_0^{\vartheta_1} y''\, d\vartheta = \Omega\, y_1''\, \vartheta_1$$

$$y_1' = y_1'' \vartheta_1$$

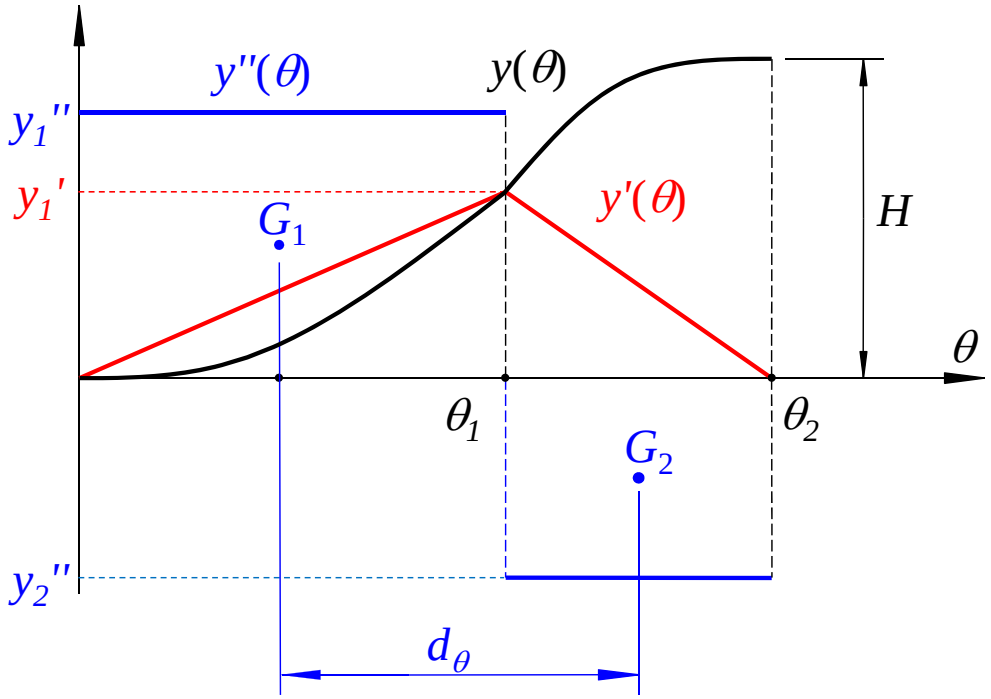
$$H = \int_0^{t_2} \dot{y}\, dt = \int_0^{\vartheta_2} y'\, d\vartheta = \frac{1}{2} y_1'\, \vartheta_2$$

$$y_1'' = \frac{2H}{\vartheta_1 \vartheta_2}$$

$$y_2'' = -\frac{2H}{\vartheta_2 (\vartheta_2 - \vartheta_1)}$$

Leggi di Moto

Legge «*parabolica*»



$$d_\theta = \frac{\theta_2 - \vartheta_1}{2} + \frac{\theta_1}{2} = \frac{\theta_2}{2}$$

$$c_v = \frac{\theta_2}{d_\theta} = 2$$

$$c_{a+} = \frac{y_1''}{H/\theta_2^2} = \frac{2}{\frac{\theta_1}{\theta_2}}$$

$$c_{a-} = \frac{|y_2''|}{H/\theta_2^2} = \frac{2}{1-\frac{\theta_1}{\theta_2}}$$

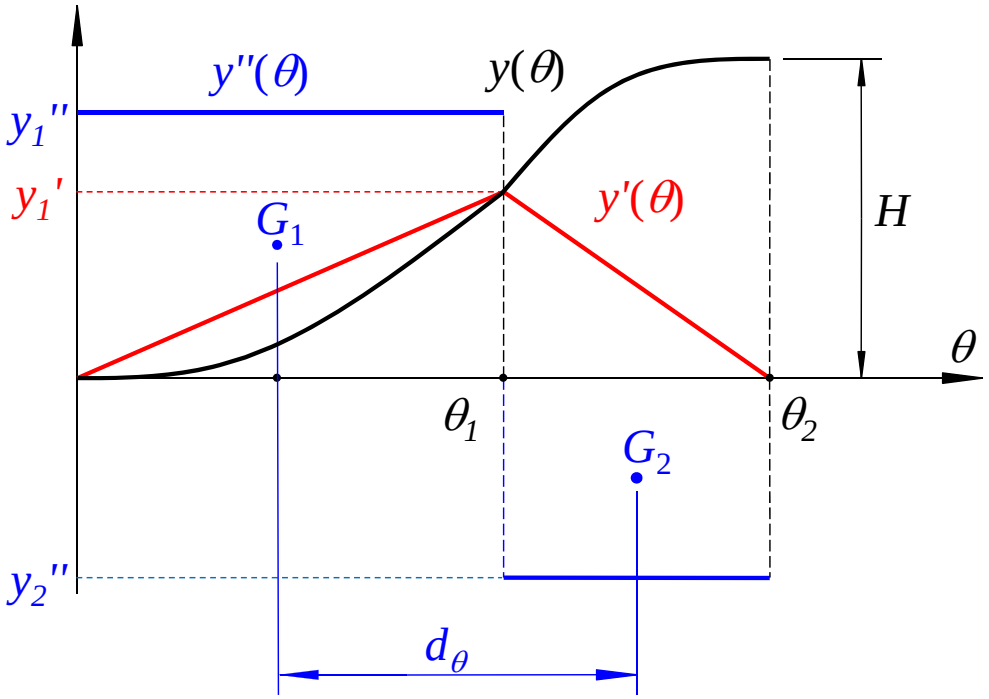
$$\theta = \vartheta_1 \rightarrow y' = y'_{\max}, y'' = y''_{\max}$$

$$c_{k^+} = c_v c_{a^+}$$

$$c_{k^+} = c_v c_{a^+}$$

Leggi di Moto

Legge parabolica



$$\theta_1 = \frac{\theta_2}{2}$$

$$c_v = 2$$

$$c_{a+} = 4$$

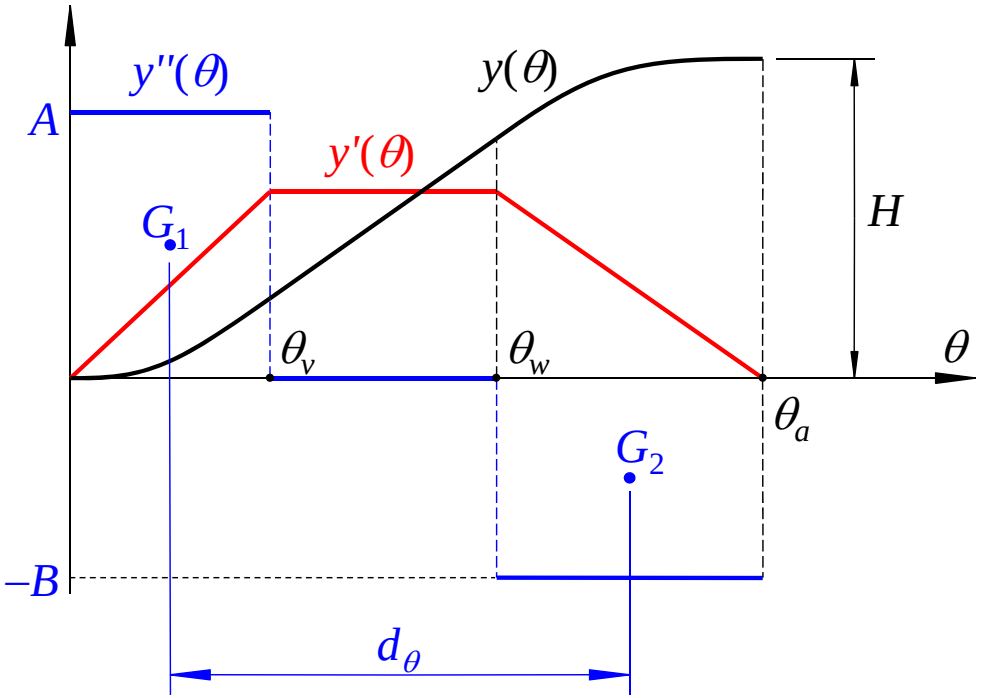
$$c_{a-} = 4$$

$$c_{k+} = 8$$

$$c_{k-} = 8$$

Leggi di Moto

Accelerazione costante (a tratti)



$$\theta_{va} = \frac{\theta_v}{\theta_a} \qquad \theta_{wa} = \frac{\theta_w}{\theta_a}$$

$$d_\theta = \frac{\theta_v}{2} + \left(\theta_w - \theta_v\right) + \frac{\theta_a - \theta_w}{2} = \frac{\theta_a + \theta_w - \theta_v}{2}$$

$$c_v = \frac{\theta_a}{d_\theta} = \theta_a \cdot \frac{2}{\theta_a + \theta_w - \theta_v} = \frac{2}{1 + \theta_{wa} - \theta_{va}}$$

$$c_{a+} = \frac{A}{H/\theta_a^2} = c_v \frac{\theta_a}{\theta_v} = \frac{c_v}{\theta_{va}}$$

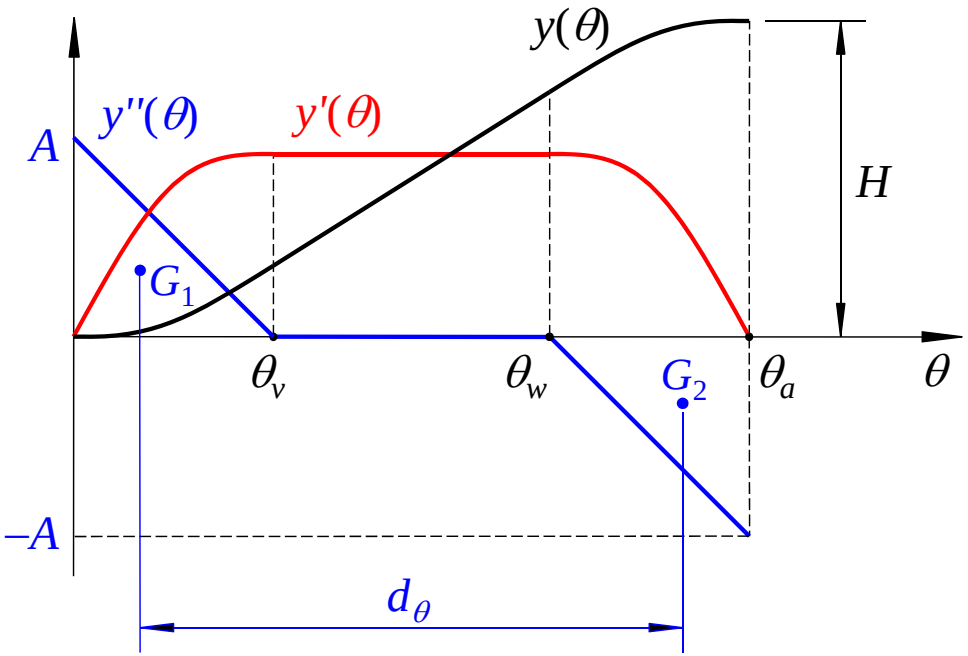
$$c_{a-} = \frac{B}{H/\theta_a^2} = c_v \frac{\theta_a}{\theta_a - \theta_w} = \frac{c_v}{1 - \theta_{wa}}$$

Il tratto ad accelerazione nulla, riduce c_v, ma innalza c_a. Ripartendo in modo uniforme i tratti ad accelerazione positiva, nulla e negativa, si ottiene un compromesso tra c_v e c_a, minimizzando c_k.

$$c_{k+} = c_v c_{a+} = \frac{c_v^2}{\theta_{va}}, \qquad c_{k-} = c_v c_{a-} = \frac{c_v^2}{1 - \theta_{wa}}$$

Leggi di Moto

Accelerazione lineare (a tratti) simmetrica



$$\theta_v = \theta_a - \theta_w \qquad \Rightarrow \qquad \theta_w = \theta_a - \theta_v$$

$$d_\theta = \frac{3\theta_a - 2\theta_v}{3}$$

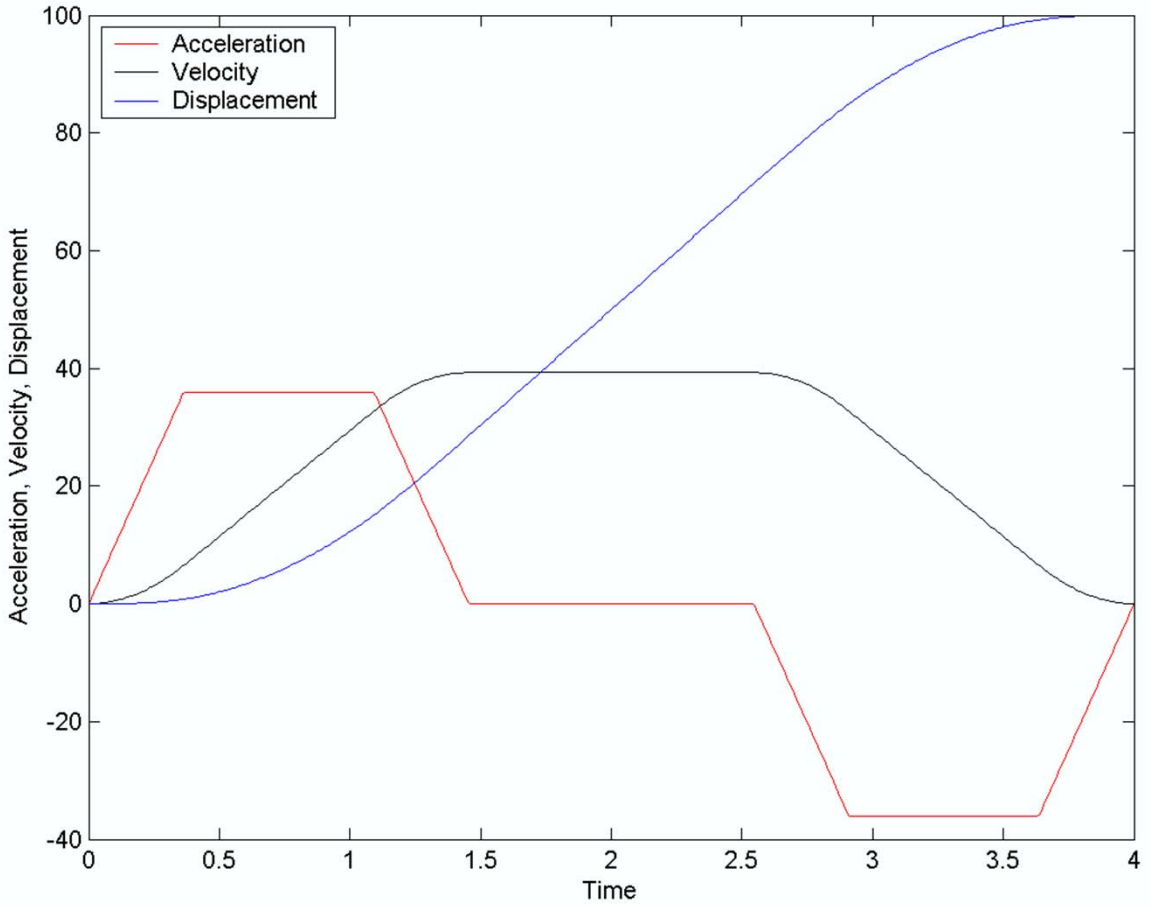
$$c_v = \frac{3}{3 - 2\theta_{va}}$$

$$c_a = c_{a+} = c_{a-} = \frac{6}{\theta_{va}\left(3 - 2\theta_{va}\right)}$$

$$c_k = c_{k+} = c_{k-} = \frac{4\sqrt{3}}{\theta_{va}\left(3 - 2\theta_{va}\right)^2}$$

Leggi di Moto

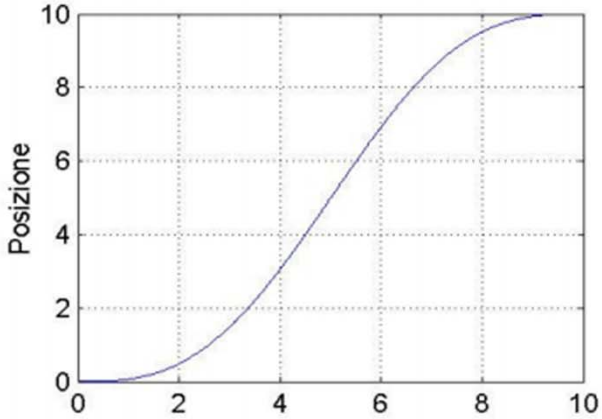
Accelerazione trapezoidale

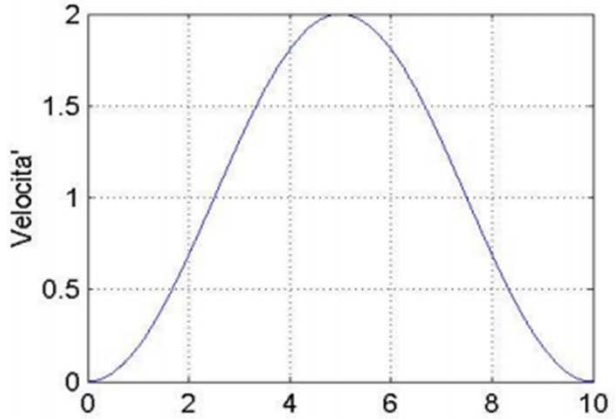


Leggi di Moto

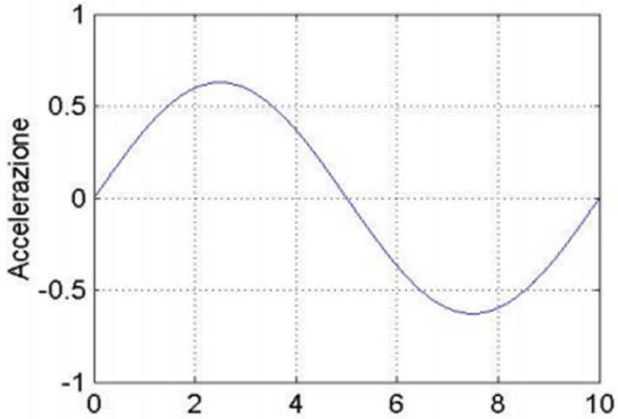
Leggi cicloidali

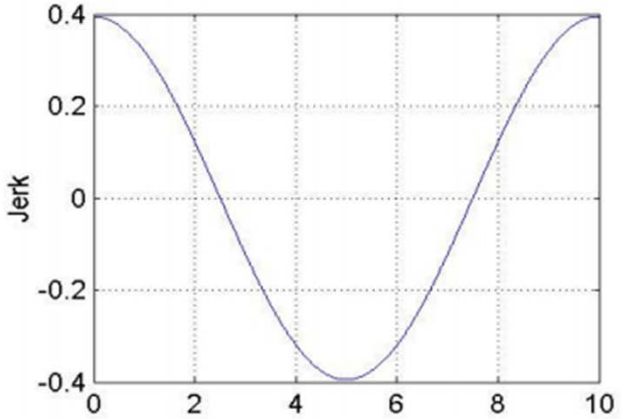
$$y = \frac{1}{\pi}\left(\pi\vartheta - \frac{1}{2}\sin(2\pi\vartheta)\right)$$





$$y'' = 2\pi\sin(2\vartheta)$$

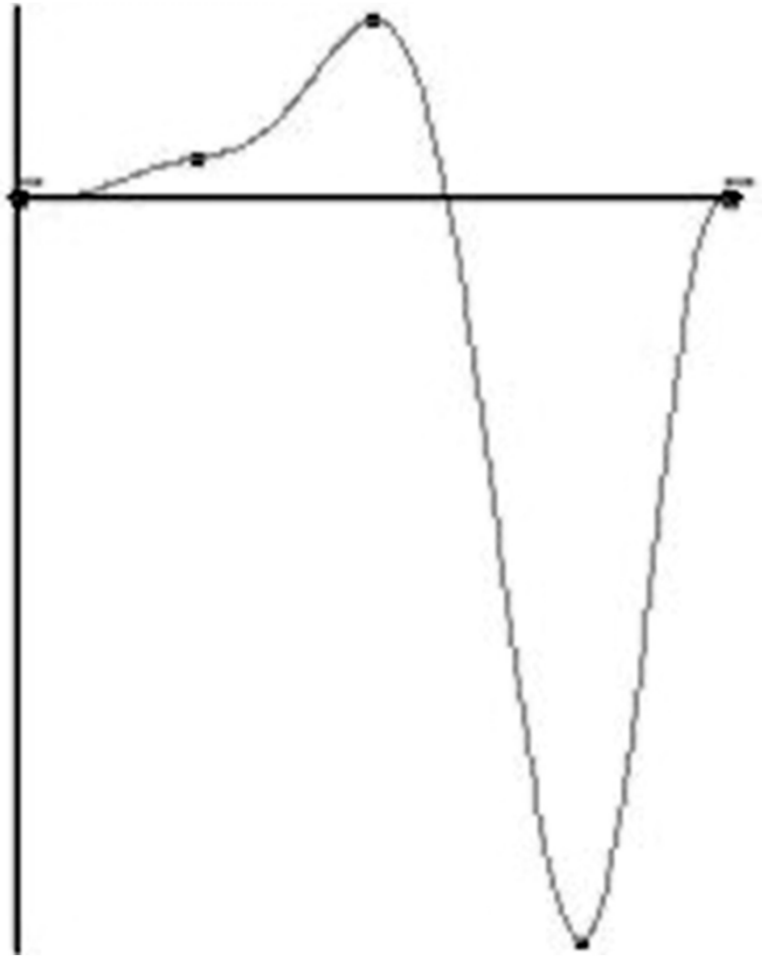




Leggi di Moto

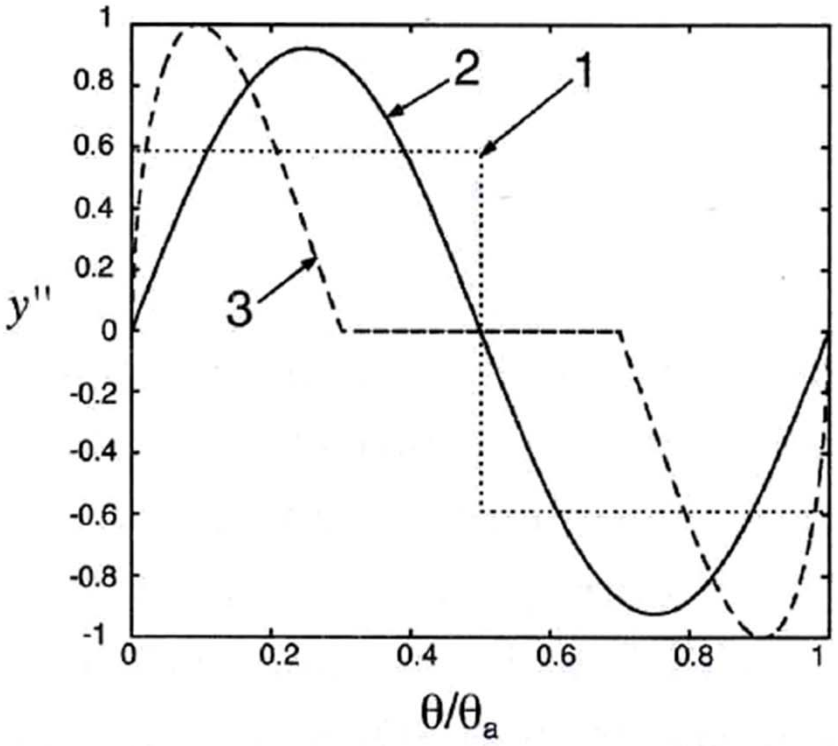
Leggi polinomiali

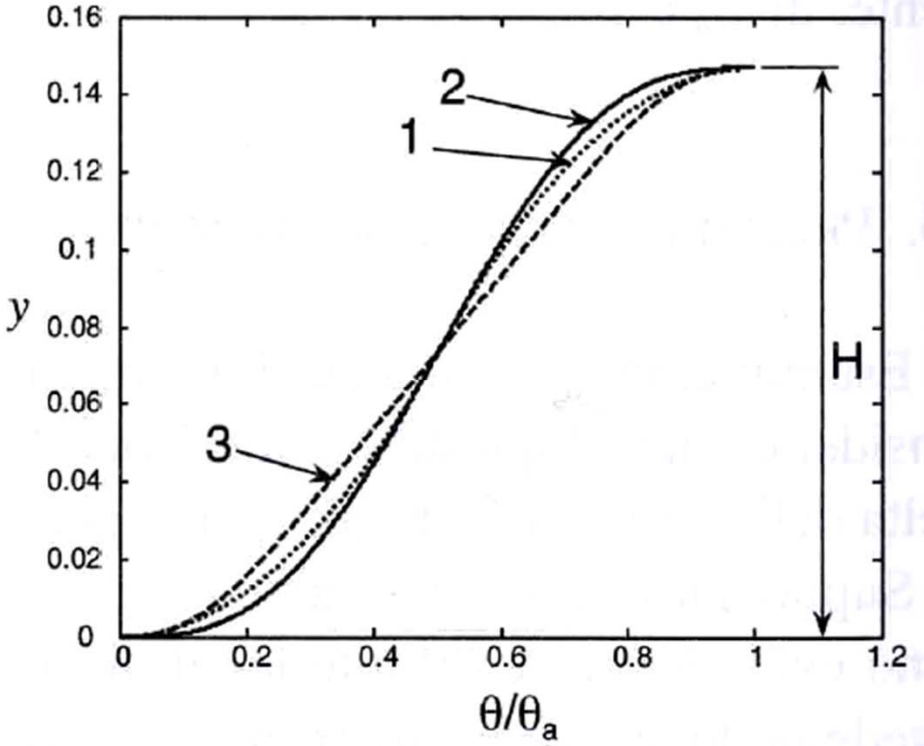
$$y_a = a_0 + a_1\vartheta + a_2\vartheta^2 + \ldots + a_n\vartheta^n$$
$$y_b = b_0 + b_1\vartheta + b_2\vartheta^2 + \ldots + b_n\vartheta^n$$
$$\ldots$$
$$y_z = z_0 + z_1\vartheta + z_2\vartheta^2 + \ldots + z_n\vartheta^n$$



Leggi di Moto

Accelerazioni vs. Spostamenti





Bibliografia

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2. Melchiorri C., *Traiettorie per azionamenti elettrici,* Progetto Leonardo, Società Editrice Esculapio, Bologna, 2000.